

Hong Kong Mathematics Olympiad (1999 – 2000)

Heat Event (Group)

香港數學競賽 (1999 – 2000)

初賽項目 (團體)

除非特別聲明，答案須用數字表達，並化至最簡。

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. 如果 a 是 $x^2 + 2x + 3 = 0$ 的根，求 $\frac{a^5 + 3a^4 + 3a^3 - a^2}{a^2 + 3}$ 的值。

If a is a root of $x^2 + 2x + 3 = 0$, find the value of $\frac{a^5 + 3a^4 + 3a^3 - a^2}{a^2 + 3}$.

2. 方程 $(\cos^2 \theta - 1)(2\cos^2 \theta - 1) = 0$ 恰有 n 個根，其中 $0^\circ < \theta < 360^\circ$ 。求 n 的值。

There are exactly n roots in the equation $(\cos^2 \theta - 1)(2\cos^2 \theta - 1) = 0$, where $0^\circ < \theta < 360^\circ$. Find the value of n .

3. 求 2004^{2006} 的個位數。

Find the unit digit of 2004^{2006} .

4. 設 $x = |y - m| + |y - 10| + |y - m - 10|$ ，其中 $0 < m < 10$ 和 $m \leq y \leq 10$ 。求 x 的最小值。

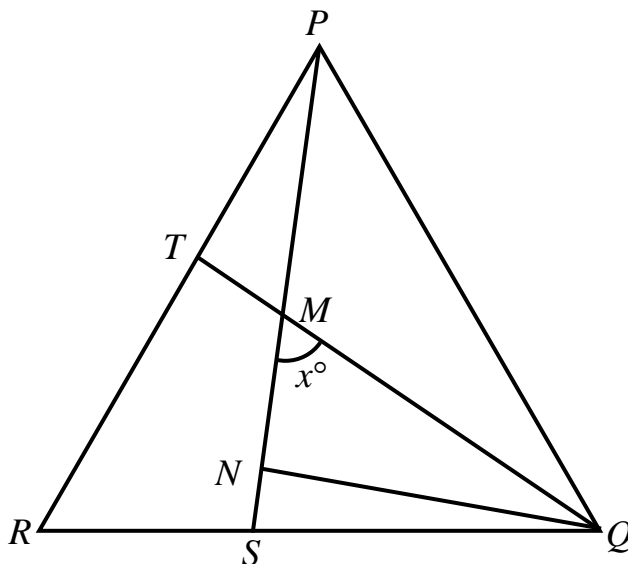
Let $x = |y - m| + |y - 10| + |y - m - 10|$, where $0 < m < 10$ and $m \leq y \leq 10$. Find the minimum value of x .

5. 有 5 個分別標上 A, B, C, D, E 的球及 5 個分別標上 A, B, C, D, E 的袋，每個袋放一個球。求恰好有 3 個球的標號與袋的標號相同的投放方法總數。

There are 5 balls with labels A, B, C, D, E respectively and there are 5 pockets with labels A, B, C, D, E respectively. A ball is put into each pocket. Find the number of ways in which exactly 3 balls have labels that match the labels on the pockets.

6. 如圖一， $\triangle PQR$ 為一等邊三角形， $PT=RS$ ； PS 、 QT 相交於 M ； QN 垂直 PS 於 N 。設 $\angle QMN = x^\circ$ ，求 x 的值。

In Figure 1, $\triangle PQR$ is an equilateral triangle, $PT=RS$; PS , QT meet at M ; and QN is perpendicular to PS at N . Let $\angle QMN = x^\circ$, find the value of x .



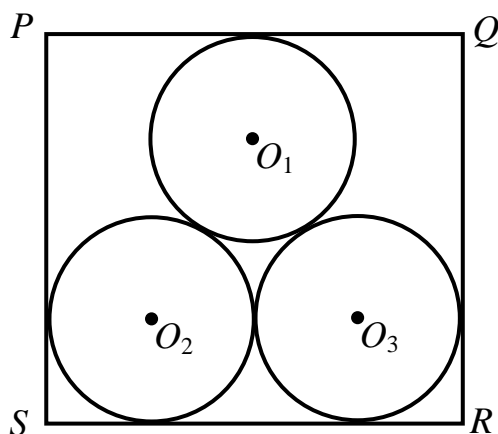
圖一

Figure 1

7. 如圖二，已知三等圓互相外切，且內切於矩形 $PQRS$ ，求 $\frac{QR}{SR}$ 的值。

(取 $\sqrt{3} = 1.7$ 及答案須準確至二個小數位)

In Figure 2, three equal circles are tangent to each other, and inscribed in rectangle $PQRS$, find the value of $\frac{QR}{SR}$. (Use $\sqrt{3} = 1.7$ and give the answer correct to 2 decimal places)



圖二

Figure 2

8. 兩個正整數之和為 29，求此兩數平方和的最小值。

The sum of two positive integers is 29, find the minimum value of the sum of their squares.

9. 設 $x = \sqrt{3+\sqrt{3}}$ 及 $y = \sqrt{3-\sqrt{3}}$ ，求 $x^2(1+y^2) + y^2$ 的值。

Let $x = \sqrt{3+\sqrt{3}}$ and $y = \sqrt{3-\sqrt{3}}$, find the value of $x^2(1+y^2) + y^2$.

10. 袋內有球 9 個，分別標上整數 1 到 9。甲從袋中隨機地抽出一個球並把它放回，乙再從同一袋中隨機地抽出一個球。把兩球上的整數相加，設 n 為該和的個位數字， $P(n)$ 為 n 出現的概率。求 n 的值使得 $P(n)$ 為最大。

There are nine balls in a pocket, each one having an integer label from 1 to 9. A draws a ball randomly from the pocket and puts it back, then B draws a ball randomly from the same pocket. Let n be the unit digit of the sum of numbers on the two balls drawn by A and B, and $P(n)$ be the probability of the occurrence of n . Find the value of n such that $P(n)$ is the maximum.